LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – APRIL 2011

# MT 4502 - MODERN ALGEBRA

 Date : 07-04-2011 Dept. No. Max. : 100 Marks

 Time : 1:00 - 4:00

 SECTION-A (10X2=20) Answer ALL the questions.

1. Let R be the set of all numbers. Define \* by x\*y=xy+1 for all x,y in R. Show that $\*$ is commutative but not associative.

 2. Define a partially ordered set and give an example.

 3. Show that the intersection of two normal subgroups is again a normal subgroup.

 4. Give an example of an abelian group which is not cyclic.

 5. Let G be the group of non-zero real numbers under multiplication. and f:G$\rightarrow $G be defined

 by f(x)=x for all x$ϵ$G. Is this map a homomorphism of G into G? Justify.

 6. If f is a homomorphism of a group G into a group G' then prove that kernel of f is a

 normal subgroup of G.

 7. Prove that an element a in a Euclidean ring R is a unit if d(a)=d(1).

 8 Let Z be the ring of integers. Give all the maximal ideals of Z.

 9. Show that every field is a principal ideal domain.

 10. Find all the units in Z[i]={x +iy/x,y $ϵ$ Z}

 SECTION-B (5X8=40)

Answer any FIVE questions.

11. Prove that a non-empty subset H of a group G is a subgroup of G if and only if HH=H and H=H-1.

12. Let H be a subgroup of a group G. Then prove that any two left coset in G are either identical or have

 no element in common.

13. Show that a subgroup N of a group G is a normal subgroup of G iff every left coset of N in G is a

 right coset of N in G.

14. Prove that any group is isomorphic to a group of permutations.

15. Prove that an ideal of the Euclidean ring R is a maximal ideal of R if and only if it is generated by a

 prime element of R.

16. Show that Q$\sqrt{2}=\{a+b\sqrt{2}, a,bϵQ\}$ is a field under the usual addition and multiplication.

 17. Let R be an Euclidean ring. Then prove that any two elements a and b in R have a greatest common

 divisor d which can be expressed by a + b.

18. Show that every finite integral domain is a field.

SECTION-C (2X20=40)

Answer Any Two

 19. a) If H and K are finite subgroups of a group G then prove that o(HK)= o(H)o( *K)/o(H*$∩K)$

 b) Prove that every subgroup of a cyclic group is cyclic. (12+8)

20. a) Prove that there is a one-one correspondence between any two left cosets of a subgroup

 H in G and thereby prove the Lagrange’s theorem.

 b) State and prove Euler’s theorem and Fermat's theorem. (10+10)

21. a) State and prove Fundamental homomorphism theorem for groups.

 b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself.

 Prove that R is a field. (12+8)

 22. a) State and prove unique factorization theorem.

 b) Let R be the ring of all real valued functions on the closed interval [0,1].

 Let M={f$ϵ$ R/ f(1/2)=0}. Show that M is a maximal ideal of R. (10+10)

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